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Leaving cert Ordinary Level

Notes

The Line

Arithmetic

Complex Numbers Probability

Sequences and Series

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Coordinate Geometry of the line.

Important Formulae.

(1) The distance formula gives the distance between two given points.

If a is the point (x_1, y_1) and b is the point (x_2, y_2)

$$|ab| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

(2) The Mid point formula:
This gives the coordinates of the mid point of the line joining (x_1, y_1) and (x_2, y_2) is given by $\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}$.

(3) The Slope formula.

The slope of the line joining the points (x_1, y_1) and (x_2, y_2) is given by the formula

$\frac{y_2 - y_1}{x_2 - x_1}$ **The symbol for slope is m.**

A word about slopes
If L_1 is parallel to line L_2 then $m_1 = m_2$ (if 2 lines are parallel their slopes are equal)

If $L_1 \perp L_2$ then $m_1 \times m_2 = -1$ (If two lines meet at right angles when you multiply their slopes you get −1).

(4) The equation of a line is an equation of the form $ax + by + c = 0$

The formula for the equation of a line is $y - y_1 = m(x - x_1)$ to use this formula we need to know two things (1) (x_1, y_2) a point on the line and (2) m the slope of the line.

(5) The formula for the area of a triangle with corners

$(0,0), (x_1, y_1), (x_2, y_2)$ is given by A = $\frac{1}{2}|x_1y_2 - y_1x_2|$ square units, the "|" symbol means the absolute value (the answer is always taken as positive).

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Coordinate Geometry of the Line
Important Formulae

(1)The distance between two points a (x_1, y_2) and b (x_2, y_2) is given by the formula

$$|ab| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

(2)The Midpoint of the line segment [ab] where a is (x_1, y_2) and b is (x_2, y_2) is given by
the formula $\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}$.

(3)The slope of the line joining a (x_1, y_1) and b(x_2, y_2) is given by the formula
$\frac{y_2 - y_1}{x_2 - x_1}$

(4)The symbol for slope is m . If two lines l_1 and l_2 are parallel $m_1 = m_2$.

(5)If line l_1 is perpendicular to line l_2 then m_1 x m_2 = - 1.

(6)To find the slope of a line perpendicular to a given line invert the given slope and change it's sign.

(7)Note also that another definition of the slope of a line is "the Tan of the angle the line makes with the positive direction of the X axis".

(8)The equation pf a line is an equation of the form $ax + by + c = 0$.

(9)The formula for the equation of a line is $y - y_1 = m(x - x_1)$.
To use this formula we must know two things (i) a point (x_1, y_1) on the line and (ii) The slope (m) of the line.

(10)The slope of the line $ax + by + c = 0$ **is –a/b (-x number over the y number)**

(11)To find the point where a line cuts the X axis let y = 0 and find x .To find the point where a line cuts the Y axis let x =0 and find y .

(12)If a point (x_1, y_1) is on the line $ax + by + c = 0$ then $ax_1 + by_1 + c = 0$. If you sub the point into the line you get 0 = 0.

Arithmetic

Ordinary Level leaving Cert

Compound Interest:

P= Principal the sum of money invested. R=Rate this is the % paid. T = time, the number of years the money is invested .I = the interest the % of the principal paid.
A is the amount = Principal + Interest.

The Compound Interest formula is $A = p(1+\frac{R}{100})^T$ A= amount Principle + Interest
P = principle, R = Rate ,T = time
Example find the compound interest on 3,500 Euros for 3 years @ 9% per annum.

$A = 3500(1+\frac{9}{100})^3 = 4532.61 \Rightarrow I = 4532.61 - 3500 = 1032.60$.

Ratio and proportion:

Method add up the ratios divide the given number by the result then multiply your answer by each of the ratios.
(1) Divide 35 in the ratio of 3:4 $3+4=7, 35 \div 7 = 5$. The ratios are 3(7) = 21, 4(7)=28.

(2) When a sum of money is divided in the ratio 7:8 the smaller part is 56 find the sum of money .The sum of the ratios is 15, therefore the smaller fraction is 7/15=56

$\Rightarrow \frac{1}{15} = 8 \Rightarrow \frac{15}{15} = 15 \times 8 = 120$.

Money: Changing from one currency to another.

If €1 = $1.12 change €200 into $ Just multiply 200 by 1.12 = $224.

Change $200 into Euro just divide $200 by 1.12 = €178.57.

Tax Credits;

Gross Tax – Tax credits = net Tax.
Example: A person earns €10, 000.
The rates of Tax are as follows Lower rate 20% on first €5, 000, and the higher rate is 42% on the rest of the income. He has tax credits of €1800 find the tax he pays.

£10,000

$£5{,}000 @ 20\% = £1{,}000$

$£5000 @ 42\% = £2100$

$Gross..Tax = £3100$

$Tax.credit = £1800$

$Nett.Tax = £1300$

Percentages;

A percentage is a fraction multiplied by 100.
Write 150g as a fraction of 2Kg.

$\frac{150}{2000} \times 100 = 7.5\%$.

Percentage Profit $= \frac{profit}{Cost\ Price} \times 100$

Complex Numbers **Ballinteer Institute,** **Leaving Cert maths**

Solve $x^2 = 25 \Rightarrow x = \pm 5$ what happens if we are asked to Solve $x^2 = -25 \Rightarrow x = \pm\sqrt{-25}$ now if you try to find the $\sqrt{-25}$ using your calculator you will get" errror 1 or error 2 "as the $\sqrt{-25}$ is not a real number to cope with this we use the following technique $\sqrt{-25} = \sqrt{25(-1)} = \sqrt{25}\sqrt{-1} = 5i$, **i** represents the $\sqrt{-1}$.

Since **i** represents the $\sqrt{-1} \Rightarrow i^2 = -1 \Rightarrow i^3 = i(i)^2 = i(-1) = -i \Rightarrow i^4 = (i^2)^2 = (-1)^2 = 1$

Numbers of the form **a+ib** are called **Complex Numbers** where a and b are Real Numbers and i is $\sqrt{-1}$.

Algebra of Complex Numbers

(1) **Addition** $(a+ib)+(c+id) = a+c+i(b+d)$ Example$3+4i+5+2i = 8+6i$

(2)**Subtraction** $(a+ib)-(c+id) = a-c+i(b-d)$Example
$4+7i-(3+2i) = 4-3+(7-2)i = 1+5i$

(3)**Multiplication of a Complex Number by a Real number** Ex : 3(4+5i)= 12 + 15i

(4) **Multiplication of a Complex Number by a Complex Number** Must remember that $i^2 = -1$
Ex : 2i(3+4i) = $6i+8i^2 = 6i-8 = -8+6i$

Ex: $(2+3i)(4-5i) = 8-10i+12i-15i^2 = 8+2i+15 = 23+2i$

Complex Conjugate Every Complex number **a + i b** has a conjugate **a-ib** .
if $z = a+ib,..\bar{z} = a-ib$

When we multiply a Complex Number by its conjugate we get the following result
(a+ib)(a-ib)$=a^2 - abi + abi - (ib)^2 = a^2 - (-1b^2) = a^2 + b^2$ **a real number** note the result is $a^2 + b^2$ (no i's) are involved :

Example ;$(3+4i)(3-4i) = 3^2 + 4^2 = 25...or(-1-i)(-1+i) = (-1)^2 + (1)^2 = 2$

Division in Complex Numbers Example
(1) Division of a Complex Number by a real number
$(6+15i) \div 3 = 2+5i$

((2) **Division of a Complex Number by a Complex Number**
(multiply above and below by the Conjugate of the bottom line)

$$(2+3i) \div (3-4i) = \frac{2+3i}{3-4i} \times \frac{3+4i}{3+4i} = \frac{6+8i+9i-12}{3^2+4^2} = \frac{-6+17i}{25} = \frac{-6}{25} + \frac{17i}{25}$$

Complex Number Equations :

Set the **Reals** = to the **Reals** and the **i's** = to the **i 's** .

Example (1) Solve for a and b
$3+4i+7+5i=a+bi:\Rightarrow 3+7=a,\Rightarrow a=10,\Rightarrow b,=4+5,a=10,b=9.$

Example (2) Solve for s and t

$s(2-i)+ti(4+2i)=1+s+ti\Rightarrow 2s-is+4ti-2t=1+s+ti\Rightarrow$

$2s-2t=1+s,\Rightarrow -s+4t=t.\Rightarrow \frac{s-2t=1}{-s+3t=0}\Rightarrow t=1,s=3.$

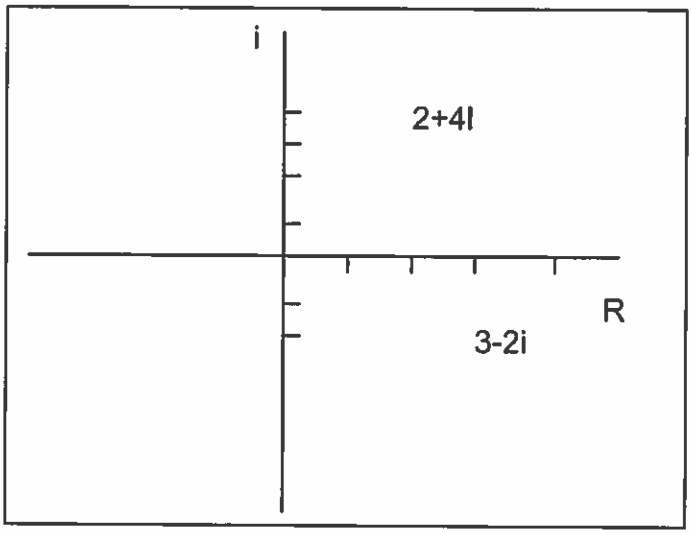
Complex Numbers and Quadratic Equations :
Solve $x^2+6x+13=0\Rightarrow x=\frac{-6\pm\sqrt{6^2-4(1)(13)}}{2}=\frac{-6\pm\sqrt{36-52}}{2}=\frac{-6\pm\sqrt{-16}}{2}=\frac{-6\pm 4i}{2}=-3\pm 4i=3+2i,3-2i$

Some important information

If the quadratic equation $x^2+bx+c=0,a,b,\in R$ has a root $x=p+iq$ then (i) $x=p-iq$ is also a root and (ii) $p+iq+p-iq=-b\Rightarrow 2p=-b$ and (iii) $(p+iq)(p-iq)=c\Rightarrow p^2+q^2=c$

Argand Diagram : A coordinate Plane for Complex Numbers , the X axis is the Real axis the Y axis is the i axis .

Plot 3-2i and 2+4i



Modulus of a Complex number.
This is the distance from (0,0) to the complex number.
The modulus of a + ib is $|a+ib|=\sqrt{a^2+b^2},\Rightarrow |3+4i|=\sqrt{3^2+4^2}=5.$

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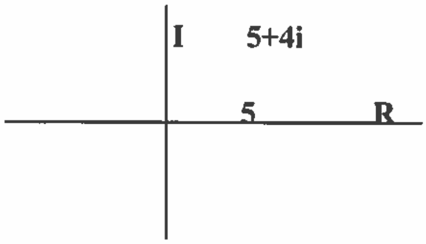
1999 Q4 Paper 1 Complex numbers
(A)
Z = 5+4i where $i^2 = -1$.
Plot (1) z, (2) z – 4i on an Argand diagram



Z = 5+ 4i, Z-4i = 5+4i-4i = 5 +0i **(10 Marks)**

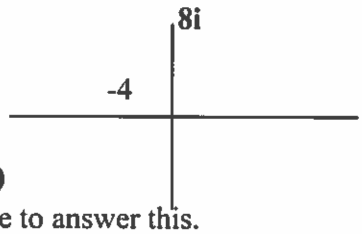
(b)(1) Let u =3 – 6i we are asked to find /3 – 6i / this the modulus of the complex number
$|x+iy| = \sqrt{x^2+y^2} \Rightarrow |3-6i| = \sqrt{3^2+(-6)^2} = \sqrt{45}$ **(10 Marks)**

(ii) **Show** $iu + \frac{u}{i} = 0$. **Multiply everything by the common denominator (i) this is the standard way to solve an equation where a bottom line is involved, this gives**
$i^2u + u = 0 \Rightarrow -1u + u = 0$ True. Another idea when you get a "solve" problem in complex numbers is just to fill in the equation with the things that you know and take it from there! Eg
$iu + \frac{u}{i} = 0 \Rightarrow i(3-6i) + \frac{3-6i}{i} = 0$, By doing this you are guaranteed the attempt mark!
You can then multiply everything by i and tidy it up. **(5 Marks)**

(iii) In this last part of part b we are asked to express $\frac{u}{u+3i}$ in the form p + q I.
Solution
First replace all the u's by 3 – 6i. This gives
$$\frac{3-6i}{3-6i+3i} = \frac{3-6i}{3-3i} = \frac{3-6i}{3-3i} \times \frac{3+3i}{3+3i} = \frac{9+9i-18i-18i^2}{9-9i+9i-9i^2} = \frac{9-9i+18}{9+9} = \frac{27-9i}{18} = \frac{3-i}{2} = 3/2 - i/2$$
(10marks)
(c) **Here we are given w = i - 2.**
Solution
We are asked to find w^2 this just means multiply w by w. this gives
$w^2 = (i-2)(i-2) = i^2 - 2i - 2i + 4 = -1 - 4i + 4 = 3 - 4i$ **(5Marks)**

In the second part we are asked to solve $kw^2 = 2w + 1 + ti$ for real k and real t.
Solution
Just replace w and w^2 in the equation. This gives k (3-4i) = 2(i – 2)+1+ ti
3k – 4ki = 2i – 4 +1 + ti. Now set reals equal to the reals and the i = i.
This gives $3k = -4+1 \Rightarrow 3k = -3 \Rightarrow k = -1$, and
$-4ki = 2i + ti \Rightarrow -4k = 2 + t \Rightarrow -4(-1) = 2 + t \Rightarrow t = 2$ **(10 marks)**
Comments:Tasks involved in this question (1)plot on an Argand diagram(a),(2)Solve a Complex Number equation(c)(3)Multiply(c)(4)Divide(b)Modulus(b)

1998 Leaving Cert Lower
Question 4 Paper 1.
(A)



Given w = 2i plot (1)..w^2...(2)..w^3 on an Argand diagram.

$w^2 = (2i)(2i) = 4i^2 = -4,..w^3 = (2i)(2i)(2i) = -4(2i) = 8i$ **(10 marks)**

Very easy same thing was asked in '99,98.'96,95 so you must be able to answer this.

(b) This part is based on the roots of a quadratic equation .We are asked to show 4 – 3i is a root of $z^2 - 8z + 25 = 0$ and find the other root, probably the easiest way is to use the roots formula

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow z = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(25)}}{2(1)} \Rightarrow \frac{8 \pm \sqrt{64-100}}{2} = \frac{8 \pm \sqrt{-36}}{2} = \frac{8 \pm \sqrt{36}\sqrt{-1}}{2}$$

$$\frac{8 \pm 6i}{2} \Rightarrow z = 3 + 4i, z = 3 - 4i.(10.Marks)$$

(b)(ii) Investigate if $|2 + 14i| = /10(1-i)\|$ here we are asked to find the modulus of two complex numbers

Solution;

$|2 + 14i| = \sqrt{2^2 + 14^2} = \sqrt{200}$, Now find the modulus of 10-10i $= \sqrt{10^2 + (-10)^2} = \sqrt{200}$.

The results are equal! **(10 marks)**

© Given u =2 – i. We are asked to write $u + \frac{1}{u}$ in the form a +bi.

Solution

Again very easy

$2 - i + \frac{1}{2-i}$, First get $\frac{1}{2-i}$ in the form a + ib as follows

$$\frac{1}{2-i} = \frac{1}{2-i} \times \frac{2+i}{2+i} = \frac{2+i}{2+2i-2i-i^2} = \frac{2+i}{5} \Rightarrow 2 - i + \frac{1}{2-i} = 2 + i + 2/5 + i/5 = 12/5 + 6i/5$$

(10 Marks)

(ii) In the last part of this question we use the information found in part (i) to solve the equation $k(u + \frac{1}{u}) + ti = 18$,

Solution

Just fill in the parts that you know $k(12/5 + 6i/5) + ti = 18$

$k(12/5) = 18 \Rightarrow 12k = 90 \Rightarrow k = 7.5,$:

$k6i/5 + ti = 0i \Rightarrow 6k + 5t = 0 \Rightarrow 6(7.5) + 5t = 0 \Rightarrow 45 = -5t \Rightarrow t = -9$ **(10 marks)**

Comment: Tasks (1)Plot includes w^3 (there must have been lots of errors with this in'97)

(2)Modulus (b)Divide(c)(3)Complex Number equation(c)(4)Roots of a quadratic(b)

Leaving Cert ordinary Level 1997.
Question 4 paper 1.
(a)
Here we are asked to get rid of the brackets and tidy it up
Solution.
3(1 +5i)+ i (3 – 2i) = 3 +15i + 3i-2(-1) = 5+ 18 i. This was worth **10 marks!**

(B)(i) Another question based on the modulus of a Complex number
In this case we are asked to find the values of **a** for which $|a+8i|=10$?
Solution
$\sqrt{a^2+8^2}=10 \Rightarrow a^2+64=100 \Rightarrow a^2=36 \Rightarrow a=\pm 6$ **(10marks)**

(ii) Given w = 4i we are asked to verify $w^3-w^2+16w-16=0$
Solution just replace w in the equation by 4i
$(4i)^3-(4i)^2+16(4i)-16=0 \Rightarrow (4i)(4i)(4i)-(4i)(4i)+64i-16=0 \Rightarrow -64i+16+64i-16=0$
true **(10 Marks)**

© In this part of the question we have to divide two complex numbers and solve an equation .
We are given z =1 +i . We are asked to find $\frac{z}{\bar{z}}=\frac{1+i}{1-i}=\frac{1+i}{1-i}\times\frac{1+i}{1+i}=\frac{1+i+i+i^2}{1+i-i-i^2}=\frac{2i}{2}=i$
(10marks)

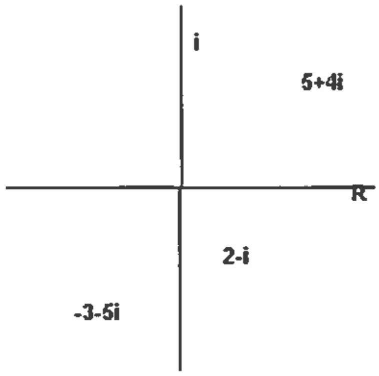
Now we are asked to use this result to Solve (very similar to '98,)
Solution :
$k\left(\frac{z}{\bar{z}}\right)+tz=-3-4i \Rightarrow ki+t(1+i)=-3-4i \Rightarrow ki+t+ti=-3-4i \Rightarrow t=-3,$ **(10 marks)**
$k+t=-4 \Rightarrow -3+k=-4 \Rightarrow t=-1$

Comments This was a very short question ,You were not asked to plot on an Argand diagram here but all the rest of the usual tasks are here. (1)Multiply(a),(2)Conjugate(c) (3)complex number equation (c)(4)modulus(b).
What was a bit unusual was the second part of (b) it had not been asked before but as you can see from above it really was an exercise in multiplying out complex numbers.

1995 Leaving Cert Ordinary Level Question 4 Paper 1

(a) Given $z_1 = 5 + 4i,..z_2 = -3 - 5i,..i^2 = -1$. We are asked to plot the following $z_1, z_2, z_1 + z_2$. this is very straightforward since we know the first two numbers already

To get $z_1 + z_2$ just add 5+4i + -3-5i = 2-i for doing this you got **10 marks!** **(2 marks for axes)**



(b) This consists of three parts

(1) We must change $w = \dfrac{1+i}{2-2i}$ into the form p+iq .

(2) Solution: We do this by multiplying above and below by the conjugate of the bottom-line this gives

$$\frac{1+i}{2-2i} \times \frac{2+2i}{2+2i} = \frac{2+2i+2i+2i^2}{4+4i-4i-4i^2} = \frac{4i}{8} = \frac{i}{2} = W$$ **(10marks)**

(2) We are asked for the modulus of $w = \sqrt{0+\frac{1}{2}}^2 = \sqrt{\frac{1}{4}} = \frac{1}{2}$.

(5 marks)

(3) We are asked to show modulus squared is equal to w multiplied by it's conjugate i.e. is

$(\frac{1}{2})^2 = \frac{i}{2} \times \frac{-i}{2} \Rightarrow \frac{1}{4} = \frac{1}{4}$ Which is true! **(5 marks)**

(3) Again three parts two of which are Complex number equations.

(1) Given u = 6 – 5i find a and b if u + ai = 2b

Solution Just replace u by 6 – 5i and set reals equal to reals and i's = to i's.

This gives 6-5i + ai = 2b $\Rightarrow 6 = 2b \Rightarrow b = 3, ai = -5i \Rightarrow a = -5$ **.10 marks**

The next part is just a slightly more complicated version of the last part; again it's a Complex number equation.

(2) Solve for real s and t. s (2-i) +ti (4+2i) = 1 + s + ti

Solution just multiply it out and set reals equal to reals and i = i.

$2s - is + 4ti + 2ti^2 = 1 + s + ti \Rightarrow 2s - is + 4ti - 2t = 1 + s + ti$

$\Rightarrow 2s - 2t = 1 + s \Rightarrow s - 2t = 1 \Rightarrow s - 2t = 1$ **(5marks)**

$\Rightarrow is + 4ti = ti \Rightarrow s + 4t = t \Rightarrow s - 3t = 0 \Rightarrow t = 1, s = 3 \Rightarrow s + it = 3 + i$

A bit long but not difficult, this type of question is a regular on LCMaths paper 1.

(3) The Last part of this question was a bit off the wall it falls into the category of ask a quadratic at all costs. You are told that a complex number Z = x + iy we are asked what type of curve is represented by $|z|^2 = |s + it|^2$

Solution

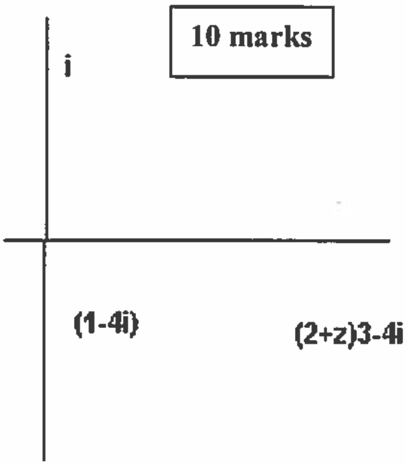
$|x + iy|^2 = |s + it|^2 \Rightarrow (\sqrt{x^2 + y^2})^2 = (\sqrt{3^2 + 1^2})^2 = x^2 + y^2 = 10$ a circle. **(5 marks)**

The way the marks are allocated clearly shows how badly parts (2) and (3) were attempted!

Comments Last bit of part c was unusual but all of the regular tasks were asked again (1)Plot(a)(2)Divide(b)(3)Complex Number equations(c)(4)Modulus (b)(5)Conjugate(b)

1996 Leaving Cert Ordinary Level paper 1 Question 4.

(a) Given z = 1 –4i as usual we are asked to plot Complex numbers on the Argand Diagram. The want us to plot z and 2 +z Z = 1 – 4i, 2 + z = 2 + 1 – 4i=3-4i.
Solution



10 marks

Part (b) Again consists of three parts this was a feature of this question up to 1996 thereafter just 2 parts.

(a) w = (1-3i)(2+i). Here we are asked to multiply out two complex numbers.

Solution

$(1-3i)(2+i) = 2+i-6i-3i^2 = 2+i-6i+3 = 5-5i = w$

(10) marks

(2) Based on the modulus and the conjugate of w.

Show $\left|w+\bar{w}\right| = \left|w-\bar{w}\right|$

Solution. Just sub in for w and it's ($\bar{w} = 5+5i$ **) conjugate to get** $|5-5i+5+5i| = \|5-5i-(5+5i)\|$

$|10| = |-10i| = 10$ **True (10 marks)**

(3) We are asked to find a if $\frac{\bar{w}}{2i} = aw$. this is a Complex number equation.

Solution Just fill in for w and $\bar{w}$ and cross multiply, then set reals equal to reals and i's = to i's. This gives

$\frac{5+5i}{2i} = a(5-5i) \Rightarrow 5+5i = 2i(a)(5-5i) \Rightarrow 5+5i = 10ai - 10ai^2 \Rightarrow 5i = 10ai \Rightarrow \frac{5}{10} = a$.

(5 marks)

© Based on a Quadratic Equation:

Given z =2 – i is a root of $z^2 + pz + q = 0$ **find p and q.**

Solution: Since 2 – i is a root then 2 + i is also a root. We know the sum of the roots is –p this gives $2-i+2+i = -p \Rightarrow 4 = -p \Rightarrow p = -4$**, and the product of the roots is q this gives** $(2-i)(2+i) = q \Rightarrow 4+2i-2i-i^2 = q \Rightarrow 5 = q$ **(15 Marks)**

Comments Contains all of the usual tasks .
(1)Plot (a) (2)Multiply (b) (3)complex equation(b). (4)modulus and conjugate (b)(5)Roots of a Quadratic(c)

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Permutations and Combinations

Factorials : n ! = n(n-1)(n-2)(-3)....................1. So 5! = 5x4x3x2x1. 7! = 7x6x5x4x3x2x1. You can find factorials using you calculator. To find 6! press 6 n!=, you will get 720.

Permutations.

This is another word for arrangements or if you like the number of ways we can rearrange a group of objects.

Definition: The number of ways of arranging n different things is n!.

Therefore the number of ways of arranging 7 different objects is 7!. At leaving cer level we can only be asked to find the number of arrangements of different objects.

Ex 1 : In how many ways can the letters of the word CAUTION be arranged. Answer 7 letters therefore 7! ways ie 5040 ways.

EX 2 In how many ways can this be done if each arrangement begins with a C, Answer 1/7 of the total will begin with a C so our answer is 1/7 of 7! = 720.

Ex 3 : In how many ways can this be done if each arrangement ends with a N, Answer again 1/7 of the total end with a n, therefore our answer is 1/7 of 7! = 720.

Ex 4 : In how many ways can this be done if each arrangement begins with a vowel. Answer 4/7 of the letters are vowels so 4/7 of the total number of arrangements will begin with a vowel ie 4/7 of 7! = 2880.

Ex 5 : In how many ways can this be done if each arrangement ends with a consonant. Answer there are 3 consonants (C,N,T) therefore 3/7 of the total will end in a consonant ie 3/7 of 7! = 2160.

Ex 6 : How many of the arrangements begin with a C and end with an N. Answer if each arrangement begins with C and ends with N, then each arrangement is of the form C - - - - - N. There are now only 5 letters to arrange these can be arranged in 5! ways.

In all the above examples we were arranging all the letters. You will sometimes be asked to find the number of ways of arranging N objects taking them R at a time.(R < N). The number of ways of arranging N objects R at a time is called ^np_r = (n)(n-1)(n-2)(n-3)..........(n - r + 1), n down r places. $^5p_3$ is 5x4x3, $^8p_3$ = 8x7x6.

Ex 7 :In how many ways can the letters a,b,c,d,e,f, be arranged taking them 4 at a time? .Answer $^6p_4$ = 6x5x4x3 =360.

Ex 8: How many 4 digit numbers can be made using the digits 1,2,3,4,5,6,7,8.if no digit is repeated Answer $^8p_4$ = 8x7x6x5 = 1680.

Combinations : Selections: Sets

These are selections, or Sets, unlike the permutations where a different arrangement means a different answer, in combinations once we have made our selection rearranging it does not give a new answer remember that combinations are sets and as you know from Junior cert the order of the elements in a set is of no importance.

Example1

Once a team is selected rearranging the players into different positions does not change the selection we still have the same players we selected in the first place.

The number of combinations of n objects taken r at a time is called" (ncr) "written ^nC_r or $\binom{n}{r}$ this is equal to ^np_r : r!

In a simplified form $^8c_3 = \frac{8x7x6}{3x2x1}$ $^{11}c_4 = \frac{11x10x9x8}{4x3x2x1}$, (11 down 4 places)/4!

Ex 9 In how many ways can a committee of 5 be chosen fom 4 girls and 3 boys ? Answer $^7c_5$. = $\frac{(7x6x5x4x3)}{5!}$.

Ex 10 In how many ways can 6 numbers be selected from 42 numbers ? Answer $^{42}c_6$.

Note $^Nc_R = {^Nc_{N-R}}$ this means $^{10}c_7 = {^{10}c_3}$, $^{15}c_{10} = {^{15}c_5}$. This can come in handy if you have no calculator !.

Ex 11 In how many ways can a committee of 5 be chosen from 4 men and 5 women if one of the men is not to be selected ? . Answer $^8c_5$.

Ex 12 In how many ways can the committee be selected if it must contain 3 women.

Answer Number of ways of selecting the women is ^5c number of ways of selecting the men is ^4c answer is

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Notes on Probability :

Some knowledge of basic probability is required for both the higher and lower leaving cert courses:

Probability is the study of random experiments .

Words :

Sample Space (S) This is the set of all possible outcomes of a given experiment .(The total number of answers that we can get when we perform a test).

Event (E): This is a set of outcomes which is a subset of the sample space .(The total number of right answers or the total number of answers that we want)

Definition

In a test there are n equally likely outcomes and let an event E consist of r of these outcomes then the probability of E is defined as follows .

$$P(E) = r/n$$

This definition can be described in various different ways (1) since both E and S are sets then the probability of e could be written as P(E) = #E/#S. (2)Or more simply the probability that we get the answer we want is the (number of right answers)/(total number of answers) .

Most leaving cert level questions on probability involve one of the following (a) Tossing of one or more coins, (b) Throwing one or more dice . (c) Picking cards from a pack . (d) Picking coloured marbles from a bag , (e) Birthdays .

Experiment 1: When a coin is tossed once what is the probability that we a head . When we toss a coin we can get two equally possible outcomes H or T so the sample space is two, the number of right answers is 1 so the probability of getting a head is 1/2.

Experiment 2 : Find the probability of getting a 4 when a die is thrown . Total number of answers is 6 (1,2,3,4,5 or6.) . Total number of right answers is 1 so the probability of getting a 4 is 1/6.

Experiment 3 : Find the probability of getting a number greater than 4 when a die is thrown . Total numbers of answers is 6, total number of right answers is 2 (5,or6), therefore the probability of getting an answer greater than 4 is 2/6 = 1/3.

Experiment 4:From a pack of 52 cards a card is drawn what is the probability that the card is a 6 . Total numbers of answers is 52 total number of right answers ,4 answers . Therefore the probability of getting a a 6 is 4/52 .

Experiment 5 : From a pack of 52 a card is drawn what is the probability of getting a picture card (Jack,Queen,King) . Total number of answers is 52 ,total number of correct answers is 12 . Therefore the probability of getting a picture card is 12/52.

Experiment 6 : From a bag containing 4 red marbles ,3 blue marbles ,and 6 yellow marbles a marble is draw find the probability that the marble is blue . Total number of answers is 13 (4 + 3 + 6) , total number of right answers is 3 . therefore the probability of getting a blue marble is 3/13 .

Experiment 7 . Find the probability that a persons birthday will fall on a Friday . Total number of answers is 7 total number of right answers is 1 . Therefore the probability of the birthday falling on a Friday is 1/7.

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Probability Theorems:

Theorem 1 : The probability of a certainty is 1.

Theorem 2 : The probability of something not happening = 1 - (the probability that it will happen). EX The probability of getting a 4 in experiment 2 is 1/6 therefore the probability of not getting a 4 is 1-1/6 = 5/6. We also use this method in problems involving "at least ".

The And /Or Theorems.

Theorem 3: The probability of event a and event b(where a is independent of b) occuring is P(a).P(b).

Theorem 4:The Probability of event a or event b (where a is independent of b) occuring is P(a) + P(b) .

Theorem 5 :The probability of event a or event b occuring where a and b are not mutually exclusive is P(a) + P(b) - P(a n b).

Mutually exclusive events ,are events when if one event occurs the other is automatically excluded . Non mutually exclusive events are events where one occuring does not automatically exclude the other . Ex find the probability of a king or a spade being drawn from a pack of 52 cards here we have the possibility of a King of spades being drawn so our answer here is 4/52 + 13/52 - 1/52 = 16/52.

Experiments involving two or more events.

Cards from a pack :

Experiment 8 : Two cards are drawn one after another from a pack of 52 the first card is not replaced Find the probability that(1) both cards are hearts
Answer total number of outcomes for first card is 52 total number of right answers is 13 so probability of first card being a heart is 13/52, since the card is not replaced the probability of the second card being a heart is 12/51 therefore the probability that both are hearts is (13/52)(12/51).

Experiment 9 :One card is drawn after another froma pack of 52,the first card is not replaced. Find the probabilitythat the first card is a king and the second card is a 10. Using the same ideas as above the probability of the first card being a king is 4/52 ,the probability of the second card being a 10 is 4/51 therefore thev probability that the first is a king and the second is a 10 is (4/52)(4/51).

Experiment 10:A person is dealt 5 cards from a pack of 52 what is the probability that they are allof the same suit .The probability that they are all hearts is (13/52)(12/51)(11/50)(10/49)(9/48) we get the same result for Clubs, Diamonds, and Spades. So our answer is 4{(13/52)(12/51)(11/50)(10/49)(9/48) } ie P(H) + P(D) + P(S) + P(C) because we require hearts or diamonds or spades or clubs .

Experiment 11:A card is taken from a pack and not replaced a second card is then withdrawn find the probability that at least 1 is a Jack . the probability that the first card is not a Jack is 48/52 ,the probability that the second card is not a Jack is 47/51, therefore the probability that we do not get a Jack is (48/52)(47/51) so the probability that at least one is a Jack is 1 - (48/52)(47/51) .

Probability:3

Coloured Marble Problems :

There are many questions based on taking coloured marbles from a container some examples of the most common are listed below.

Experiment : 12 A box contains 8 Red, 3 White, and 9 Blue marbles if 3 are taken out with no replacement find the probability that they are all white ?.
Answer : The probability that the first is white is 3/20,the probability that the second is white is 2/19, the probability that the third is white is 1/18, therefore that the first and the second and the third are white is (3/20)(2/19)(1/18).

Experiment : 13 . Find the probability that none of the three are white . If the first is not white the probability of this happening is 17/20,(because there are 17 non white marbles)
Answer (17/20)(16/19)(15/18).

Experiment : 14. Find the probability that at least one is white .
Answer :1 minus the probability that none are white which is 1 -{(17/20)(16/19)(15/18)}.

Experiment : 15 . Find the probability that the first two of the marbles are red and that the third one is white .
Answer (8/20)(7/19)(3/18).

Experiment : 16 Find the probability that two are red and one is white . This question is slightly different as the order in which we get the red marbles is not important , We now have the following situation RRW or RWR or WRR the probability for each of these is (8/20)(7/19)(3/18) so my answer is 3{(8/20)(7/19)(3/18). ie p(E1) + p(E2) + P(E3).

1994 Sample :A drawer contains 5 red and x blue biros. One is drawn at random and not replaced . Another is then drawn at random . If the probability that both are blue is 1/6 how many blue biros are in the drawer .
Answer : the probability that the first is blue is x/(x+5), the probability that the second is blue is (x - 1)/(x + 4) . So the probability that both are blue is{ (x)/(x +5) .(x - 1)/(x + 4)} = 1/6. Solving for x we get x = 4 .

1994 Sample :Of 100 tickets in a raffel 40 are Red, 30 are Blue ,and 30 are Green. The winning ticket is drawn at random what is the probability that : (1) it is Red Answer 30/100. (2) that it is not Blue Answer 70/100.
A ticket is drawn and then replaced . If three such tickets are drawn find the probability that at least are red . The probability that two are red is 3{ (40/100)(40/100)(60/100)}. The probability that three. are red is (40/100)(40/100)(40/100). therefore the probability that at least two are red is 3{(40/100)(40/100)(60/100) } + {(40/100)(40/100)(40/100)}. = .352. or we could find the probability that none are red or the probability that 1 is red and take this away from 1.

Every Red Ticket is even numbered while every blue ticket is odd numbered and of the green tickets 20 are even numbered and 10 are odd numbered. Find the probability that the winning ticket is green or even numbered .
Answer : The probability that the winning ticket is even numbered is 60/100, the probability that it is green and odd is 10/100 so the probability of green or even is .6 + .1 = .7

Ballinteer Institute   Arithmetic and Geometric Series   Leaving Cert Maths

This topic is Examined on paper 1 of the leaving Cert   H.

Words :

A Sequence is a set of numbers or letters seperated by commas eg , 1,3,5,7,9 .

A Series is a set of numbers or letters seperated by + signs eg 1+3+5+7+9 .

The elements of a sequence or a series are called the Terms of the sequence or Series

In the Series 1 + 3+5+7+9 the first term is 1,the second is 3 etc,

ie $T_1 = 1, T_2 = 3, T_3 = 5, T_4 = 7.$

General Term T_n this is an expression in n which describes every term of a particular series.

The general term of the series 1+3+5+7+9...is $T_n = 2n - 1$, substituting for n gives any term of the Series ex n = 2 $T_2 = 2(2) - 1 = 3$,

Using T_n ,

(1) to find a particular term of the series if $T_n = 2n - 1$ find the 20th term $T_{20} = 2(20) - 1 = 39$.

(2) To find which term of the above series is a particular number

Example find which term of the series is 599 , method set $T_n = 599$ and solve for n 2n-1=599
$2n = 600 \Rightarrow n = 300 \Rightarrow T_{300} = 599$.

The Sum of the first n terms of a series is called the Sn of that series(this is a formula in n which gives you what you get when you add up the first n terms of that series) . In the series 1+3+5+7+9 $S_1 = 1, S_2 = 4, S_3 = 9,$.

Types of series Type (1) Arithmetic series/sequence :Linear series

In this type of Series a constant called the common difference (d) is added to each term to give the next term .

Examples of Typical Arithmetic Series .1+3+5+7+9......2+5+8+11.......,. 10+6+2-2-6-........ .

Notice in all these Series $T_2 - T_1 = T_3 - T_2 = d$.

The first term of a series T_1 is usually represented by the letter a .

$T_2 = a + d, T_3 = a + 2d, T_4 = a + 3d$ **etc notice every term contains an *a* and the number of *d* is one less than the number of the term.**

The formula for the Tn of an Arithmetic series is Tn = a + (n-1)d .

This formula is used (1) to find Tn of a particular series

Example 1 given the Arithmetic Series 1+4+7+10, a = 1 d = $T_2 - T_1 = 3$.

this gives Tn = a + (n - 1)d , Tn = 1+(n-1)3 = 3n - 2 .

Example 2: If T_3 of an Arithmetic series is 15 and $T_8 = 45$ find a,d,Tn and T_{30}.

This gives a + (3-1)d=15, and a + (8-1) d=45 giving

$$a + 2d = 15$$

$$a + 7d = 45 \Rightarrow 5d = 30 \Rightarrow d = 6,..a = 3.$$

This gives Tn =3 + (n-1)6=6n-3, T_{30}= 6(30)- 3 = 177

TheSum of the first n terms of an Arithmetic Series is called Sn the formula which gives the Sn of a Series is

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

Example find Sn and S_{30} of the Series 3+8+13+..

a = 3 , d = 5 : $Sn = \frac{n}{2}\{2(3) + (n-1)5\} = \frac{n}{2}\{5n + 1\}$, S_{30}=30/2{5(30)+1} 15(151)=2265

Example 3: Questions based on Sn . Remember Sn gives the sum of the first n terms of the given Series also remember that $S_1 = T_1,.S_2 = T_1 + T_2, S_3 = T_1 + T_2 + T_3$etc.

Example 4:

If Sn of an Arithmetic series is $n^2 + 3n$ find

$S_1, S_2, a, d, Tn...S_1 = 1+3(1) = 4 = T_1 = a, S_2 = 2^2 + 3(2) = 10 = T_1 + T_2 \Rightarrow 4 + T_2 = 10$

$\Rightarrow T_2 = 6, d = T_2 - T_1 \Rightarrow d = 6-4 = 2, \Rightarrow Tn = 4+(n-1)2 = 2n+2$

Example 5:

If a , -5 , b , 7 are the first 4 terms of an Arithmetic series find a,b, and T_5.

We know $T_2 = -5, \Rightarrow a + d = -5, T_4 = 7 \Rightarrow a + 3d = 7, \Rightarrow 2d = 12 \Rightarrow d = 6, a = -11$

a = -11, b = 1, T_5 = 12 .

Example 6:

T_{10} of an Arithmetic Series is 19 , S_{10} =35 find a and d

$$T_{10} = 19 \Rightarrow a + 9d = 19,$$

$$.S_{10} = 55 \Rightarrow \frac{10}{2}\{2a + 9d\} = 55 \Rightarrow 10a + 45d = 55.(\div 5) \Rightarrow 2a + 9d = 11$$

$$\Rightarrow a = -8, d = 3$$

Quadratic Sequences

$$T_n = an^2 + bn + c$$

$T_1 = a + b + c, T_2 = a(2)^2 + b(2) + c = T_2 = 4a + 2b + c, T_3 = a(3)^2 + 3b + c \rightarrow T_3 = 9a + 3b + c$

If we find the differences between the terms $T_2 - T_1 = 3a + b = d_1$ $T_3 - T_2 = 5a + b = d_1$

Now find the 2nd difference $5a + b - (3a + b) = 2a$ .For all quadratic sequences the 2nd differences are all 2a.We can use this fact if we know T_1, T_2, T_3, to find a,b,c

Give the sequence 2,9,22,41,66 Prove the sequence is Quadratic and find general term .

First differences are 7,13,19,25.The 2nd differences are 6,6,6,6 since the 2nd differences are all 6 the sequence is quadratic.From above we know that $2a = 6 \rightarrow a = 3$ .We know

$3a + b = 7(T_2 - T_1) \rightarrow 3(3) + b = 7 \rightarrow b = -2$, $3 - 2 + c = 2 \rightarrow c = 1$ $T_n = 3n^2 + -2n + 1$

Geometric series :(GP) (exponential)

In this type of series each term is multiplied by a constant to give the next term .
This constant is called the common Ratio symbol r .

Typical geometric series are 1+3+9+27+..., 1/2 +1+2+4+...

Notice that in both Series $\frac{T_2}{T_1} = \frac{T_3}{T_2} = r$.

This is a very important rule as Geometric series are more difficult to handle than arithmetic Series as they always involve powers of r .

The standard GP is of the form $a + ar + ar^2 + ar^3 + ar^4 + \ldots\ldots, Tn = ar^{n-1}$.
Notice every term contains an "a " and ,an r to a power which is 1 less than the number of the term ie $T_3 = ar^2, T_5 = ar^4 \ldots T_{20} = ar^{19}$.

Example 1: if 2, x , 18 are three terms of a GP find x, In all GP's $\frac{T_2}{T_1} = \frac{T_3}{T_2}, \ldots\ldots$

$$\frac{x}{2} = \frac{18}{x} \Rightarrow x^2 = 36 \Rightarrow x \pm 6.$$

Example 2: In a GP $T_3 = 3, T_6 = 24$**.Find The first term and the common ratio .**

$$\frac{ar^5}{ar^2} = \frac{24}{3} \Rightarrow r^3 = 8, \Rightarrow r = 2, \Rightarrow a(2)^2 = 3 \Rightarrow 4a = 3 \Rightarrow a = \frac{3}{4}.$$

The Sum of the first n terms of a GP is $Sn = a(\frac{1-r^n}{1-r}),..r<1...Sn = a\left[\frac{r^n-1}{r-1}\right],..r>1.$

Note these two formulae are the same the 2nd formula is just the first multiplied above and below by -1 .

Example 3:

Find Sn and S_{20} **of the Series** $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots..$**, a =1, r = 1/2**

$$Sn = 1\left(\frac{1-(\frac{1}{2})^n}{1-\frac{1}{2}}\right) = \left(\frac{1-(\frac{1}{2})^n}{\frac{1}{2}}\right) = 2(1-(\tfrac{1}{2})^n), \Rightarrow S_{20} = 2(1-(\tfrac{1}{2})^{20}.$$

Example 4: $Tn = 3^{n-1}$ **Find a and r and Sn . a =** T_1 =

$$3^{1-1} = 3^0 = 1 = a.T_2 = 3^{2-1} = 3, r = \frac{T_2}{T_1} = \frac{3}{1} = 3, Sn = a\left(\frac{r^n-1}{r-1}\right) = 1\left(\frac{3^n-1}{3-1}\right) = \frac{3^n-1}{2}.$$

Investigate if 2Sn - Tn = 2Tn - 1 this is the same as show 2Sn = 3Tn - 1
$2Sn = 3^n - 1, ..3Tn - 1 = 3(3^{n-1}) - 1 = 3^n - 1 = 2Sn$

The Compound interest formula :
The words
Principal (P) = sum of money Invested . Rate (R) the % of the principal paid as interest, Time (n) the number of years that the money is invested .
Amount (A) = Principal + Interest .

The formula $A = P(1+R/100)^n$ **.**
Gives the Amount that The principal P will become in n years at R% .

Ex 1 Find what £1500 will amount to in 6 years at 8% .

A = 1500(1+8/100)^6= 1500(1.08)^6 . A = £2380.31.

Example 2:
£2,500 is invested for 3 years the rate on year 1 was 4%, year 2 was 3%. If the amount at the end of the 3rd year was £2744.95 find the rate in the 3rd year .
2500(1.03)(1.04) = £2678 = amount at end of year 2 . Interest on year 3 is 66.95 = (c2744.95 – 2678)
as a %, $\frac{66.95}{2678} \times 100 = 2.5\%$

Or $2744.95 = 2678(1+\frac{R}{100})^1 \Rightarrow \frac{2744.95}{2678} = (1+\frac{r}{100}) = 1.025 \Rightarrow R = 2.5$