www.tuition.ie

email:john@tuition.ie

tel 0872563032

# Algebra Notes

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John Brennan ©

**Rules for Algebra** **www.leavingcertsolutions.com**

**Rule 1 Addition** of two Numbers with **the same sign** The result is the sum of the numbers with the given sign ( the sign of the two numbers ) .

When **we add two plus numbers we get a plus number** **Ex : 3+6=9.**

When **we add two minus numbers we get a minus number** **Ex : -5 + - 6 = - 11 .**

Example 5 + 6 + 8 = 19 . Example -5 + - 8 + - 6 = - 19 .

**Rule 2 : Addition** of two numbers with **different signs** the result is the difference **between the numbers** with the **sign of the" bigger number "** .

Ex 6 + - 9 = - 3 : Ex 19 + -5 = 14 . Ex -15 + 13 = - 2 .

Note Ex : 14 - 9 is the same as 14 + - 9 = 5 . and - 7 - 5 is really -7 + - 5 = - 12

Example simplify - 6 + 9 - 7 + 3 + 6 - 8 = - 6 - 7 - 8 + 9 + 3 + 6 = - 21 + 18 = - 3 .

Note we add the minus numbers ,we add the plus numbers then tidy it up .

**Rule 3 : Multiplication and signs :**

**The basic rules for multiplication and signs are as follows**

**(Minus ) x ( Minus ) = Plus . ( Minus ) x ( Plus ) = Minus , (Plus ) x (Plus ) = Plus .**

This can be expressed as follows that (a) when we **multiply** two numbers with the **same sign** we get **plus** ( b) when we **multiply** two numbers with **difference signs** we get a **minus** .

Example : - 7 x - 5 = + 35 , -7 x 5 = -35 . Example - 7. x - 4 x 3 = 28 x 3 = 84 .

Example : - 5 x - 6 x - 2 = 30 . x. - 2 = - 60 . Note if we are asked to **multiply three or more numbers** remember you can only **multiply two numbers at a time** .

Example - 4 x -3 x 5 x -2 = 12 x 5 x - 2 = 60 . x - 2 = - 120 .

**Rule 4 : Letters**

Letters are often used in place of numbers , letters are often referred to as variables or unknowns . Rules 1,2,3 above apply to all calculations with letters , we have to remember one further rule which applies to letters that is **we can only add things together that are the same** ie we can only add x's to x's and y's to y's etc . Note also 5a means 5 times a .

Example 5a - 7a = - 2a , Example - 3x + 7x = 4x, Example 5a + 6b - 4a - 11b ,= 5a - 4a + 6b - 11b = a - 5b .

Multiplying letters by letters : $x \,.x = x^2 .:..x.x.x = x^3 , (3a)(2b) = 6ab, ..(2x)(-3x) = -6x^2$

**Rule 5 : Brackets :**

**(a) A number immediatly outside a bracket means that everything inside the bracket gets multiplied by that number** Ex $4(5) = 20, ...4(-5) = -20, ...-4(-5) = 20.$

Ex $5(3a + 2b - 4c) = 15a + 10b - 20c$

**(b)Minus sign outside a bracket will change all the signs inside the brackets when the brackets are removed** : ex $-(2a - 3b) = -2a + 3b, ...-(-4x + 5y - 2z) = 4x - 5y + 2z.$

**(c)Minus number outside a bracket does two things (i) everything inside the bracket gets multiplied by the number and (ii) all the signs change**

$-4(2a - 3b) = -8a + 12b, ...-7(-2b - 3c + 4d) = 14b + 21c - 28c.]$

$-4x(3x + 2y - 5z) = -12x^2 - 8xy + 20xz$

**Brackets by brackets :**

**Multiply everything in the second bracket by everything in the first bracket and tidy it up**

$(x + 2y)(x + 3y) = x^2 + 3xy + 2xy + 6y^2 = x^2 + 5xy + 6y^2$

$(3x - 4y)(2x + 5y) = 6x^2 + 15xy - 8xy - 20y^2 = 6x^2 + 7xy - 20y^2$

**Mathematical Words :**

**The following words appear frequently in Mathematics Exam Papers .**

(1) **Expression :**

This is a collection of letters and numbers : eg $(4x + 3y)(5x-7y)$ : $7x^2+4xy-6$ .

**Words** associated with expressions.

**Simplify** : Means get rid of the brackets and tidy it up :

ex Simplify $(4x+3y)(2x+5y) = 8x^2 + 20xy + 6xy + 15y^2 = 8x^2 + 26xy + 15y^2$

**Factors** : The factors of of an expression are two or more things which when multiplied together give that expression

.Example the factors of 14 are 7 and 2 because 7 x 2 =14

**Factorise** : means find the factors of : Example factorise $15bc - 3c^2 = 3c(5b - c)$

(2) **Equation :**

This is an expression which contains an equals sign : All equations contain an equals sign .

$7x- 5y = 10$ : $4x^2-6x+5 = 0$, $3x+2y = 4$

$4x- 5y = 10.$

**Word** associated with equations

**Solve** : Means find the value(s) of the letter that makes the equation true .

Example Solve $3(x+4) - 2(x-3) = 10 \Rightarrow 3x + 12 - 2x + 6 = 10 \Rightarrow x = 10 - 12 - 6 \Rightarrow x = -8.$

Types of equation (1) Linear equation one variable Example 3x-4=10

(2)**Linear equations 2 variables** $\begin{matrix} 3x-5y=7 \\ 4x+2y=8 \end{matrix}$ these are called **simultaneous equations**

(3)**Quadratic equations** : Equations of the form $ax^2 + bx + c = 0$ are called quadratic equations:

the **solutions** of a quadratic equation are called the **roots** of the equation , all quadratic equations have two roots!

$3x^2 - 4x + 2 = 0$ is a quadratic equation

**Inequality** : This is an expression which contains one of the following symbols $\leq, <, \geq, >$

**Word associated with Inequalities Solve;**

**Example Solve** $3x + 5 \leq 2$

**Verify** : this means show that the information you are given is true

**Verify if**

$a = (x + y), and\, b = (x - y)..then..a^2 + b^2 = 2x^2 + 2y^2$

$a^2 = (x+y)(x+y) = x^2 + 2xy + y^2, b^2 = (x-y)(x-y) = x^2 - 2xy + y^2 \Rightarrow a^2 + b^2 = 2x^2 + 2y^2$

**Calculate means work out ( using maths) the value of** : Example Calculate

$2\frac{1}{4}\%\,of\,£164: 1\% = £1.64, \Rightarrow 2\% = £3.28. \frac{1}{4}\% = £0.41 \Rightarrow 2\frac{1}{4}\% = £3.69....or\,\frac{£164}{100}(2.25)$

**Evaluate : (normally applies to a given expression) we are asked to simplify and find the value of .**

**Example Evaluate** $\sqrt{\frac{7}{.5} + (2.5)^2} = \sqrt{14 + 6.25} = \sqrt{20.25} = 4.5$

**Words associated with fractions :**

**Numerator** : the top half of the fraction, **Denominator** :the bottom half of the fraction.

**Common denominator :** this is the smallest number that the bottom halves of two or more fractions will divide.

Example the **common denominator** of $\frac{4}{3} + \frac{3}{5} + \frac{7}{2}$ is 30 as it is the smallest number that can be divided by 2,3, and 5 .

**Lowest common multiple (LCM)** is another name for the smallest number that a given set of numbers will divide .

**Highest Common factor (HCF)** : this is the biggest factor a set of numbers have in common example the highest common factor of 15,25 and 30 is 5 : the HCF of 8,12,and 28 is 4 .

**Words Associated with graphs :**
**Plot** this means place on a coordinate plane some given points .

**Estimate** (from your graph ) read answers from your graph .

**Types of Numbers and their Symbols**
(1) **Natural Numbers** Symbol **N** This is the set of positive whole numbers .
N =(0,1,2,3,4,.....)
On the numbers line we use" dots " to indicate Natural numbers 0 1 2 3 4 5 6

(2) **Integers** Symbol **Z** This is the Set of positive and negative whole numbers Z={-3,-2,-10,1,2,3}
on the number line we show the integers as "dots" -3 -2 -1 0 1 2 3 4

(3)**Rational Numbers** Symbol **Q** : This is the set of numbers that can be written as a fraction
Most numbers can be written as a fraction eg $3=\frac{3}{1};1.75=\frac{175}{100}:..,.7=.77777777=\frac{7}{9},..-.5=-\frac{5}{10}$

In general you will not be asked to put rational numbers on the number line .

**(4)Irrational Numbers** Symbol **Ir** : Numbers that cannot be written as a fraction :
The only numbers that cannot be written as a fraction are numbers such as $\Pi$ (a non repeating non terminating decimal) or surds such as $\sqrt{2},\sqrt{3},\sqrt{5}$.

**(5)Real numbers** Symbol **R** : This is the set which includes all of the above types of numbers On the number line we use a thick line to show real numbers -2 -1 0 1 2 3 4

**(6)Complex numbers** : These are numbers of the form x+yi where x and y are Real numbers and $(i)=\sqrt{-1}$ complex numbers are plotted on an Argand diagram.

**Inequality symbols**
In general read all information **from left to right**
>" is greater than ", $x > y$ x is greater than y . <" is less than ",$a < b$ a is less than b
$\leq$ "is less than or equal to" $x \leq k$ x is less than or equal to k .:$\geq$ "is greater than or equal to"
$p \geq h$ p is greater than or equal to h .
**Compound inequalities** $a \leq x \leq b$ read from the **middle to the left** then from the **middle to the right** the above inequality reads " x is greater than or equal to a and x is less than or equal to b "

**Symbols and Sets**
:$\in$ "is an **element** of "belongs to a particular set $\notin$ is not an element of .

. $\cup$. **Union Au B** means the set of all the elements of A and B with none repeated

$\cap$ **intersection** , A $\cap$ B means the set of all the elements that A and B have in common .
$\cap$
$\subset$ **"is a subset of "** :A$\subset$ B means that A is a subset of B , A contains some or all of the elements of B but no other elements .$\not\subset$ means **is not a subset** .

# The **cardinal number** of a set , this indicates the number of elements in the set .

**More Symbols :** $\perp$ means is **perpendicular to** : A $\perp$ B means A and B meet at right angles .

$//^{o}$ means is **parallel** to . $\Rightarrow$ means implies :

$\angle$**abc** means angle abc . $\overset{\wedge}{\mathbf{a}}\mathbf{bc}$ also means angle abc

$|\angle \mathbf{abc}|$ means the size of the angle ,

The symbol $|\ |$ in general means the **size of** or **the measure of** or the **lenght of** , but another meaning is the **absolute value** of meaning the **positive value** of something

$|-3| = 3$.

**Function notation :** A function is a rule which is shown as an expression in x ;

Ex $f(x) = x + 4$ this states that f is a function which turns x into x + 4 ,(4 is added to every x ) so f(3) = 3+ 4, f(7) = 7 + 4 .

The way functions are shown can vary the same function can be shown as follows

$\mathbf{f:x \to x + 4,...or..f:R \to R:x \to x + 4}$

The first R is **the domain** of the function these are the numbers that can be used in the function .

The second R is the **Codomain** of the function these are the numbers that can come out of the function .

Functions are often referred as maps and are said to Map (connect up ) the elements of the Domain with those of the Codomain .

But the most common notation is f(x) equals .

# Simultaneous Equations

**Linear Equations two variables :**

The most common type of simultaneous equations are two equations with two variables

Example 1 $\begin{matrix} 3x+4y=7 \\ 2x-y=1. \end{matrix}$

To solve this type of simultaneous equation . You eliminate one of the letters by **adding** the two equations this leaves you with one equation and one letter , solve this equation .

Then use your answer to find the second letter . For one of the letters to be **eliminated** by **addition** you must have the **same amount** of the letter on both lines and the **signs must be didfferent .**

Remember we eliminate one of the letters by addition only .

**Example 1** Solve $\begin{matrix} x+y=5 \\ x-y=3 \end{matrix}$ Notice that the y's have the same amount and the signs are different

$\begin{matrix} x+y=5 \\ x-y=3 \end{matrix}\Big\rangle add$

$,2x=8 \Rightarrow x=4$

**now find y** we know x+ y = 5 but x = 4 , 4 + y = 5 , y = 1.

**Example 2** : Solve $\begin{matrix} 3x+2y=11 \\ 2x-y=5 \end{matrix}$ notice in this case that the y's have opposite signs ,

if we can now organise that the **y's** have the same amounts then the **y's** will be eliminated when we add the equations .

So multiply the bottom line by 2 as follows $\begin{matrix} 3x+2y=11 \\ 2x-y=5.x2 \end{matrix}$ this gives $\begin{matrix} 3x+2y=11 \\ 4x-2y=10 \end{matrix}$ add to get $7x=21 \Rightarrow x=3$

now use this value of x to find y . We know 3x + 2y = 11 but x = 3 , 3(3) + 2y = 11 ,9 + 2y = 11 , 2y = 2, y = 1 .The solution is (3,1)

Example 3 : Solve $\begin{matrix} 3x+2y=7 \\ 2x+y=5 \end{matrix}$ notice in this case neither of the two requirements are available

so to get the equations into the correct shape just multiply the bottom line by -2

$\left.\begin{matrix} 3x+2y=7 \\ 2x+y=5.x-2 \end{matrix}\right) = \begin{matrix} 3x+2y=7 \\ -4x-2y=-10 \end{matrix}\Big\rangle add \Rightarrow -x=-3 \Rightarrow x=3$ now find y , we know 2x + y = 5 ,but x = 3 2(3) + y = 5 , 6 + y = 5 , y = - 1 .

Note it dose not matter which equation we use to find the second letter but the safest one to use is the one with least number of minus signs.

Example 4: $\begin{matrix} 2x+3y=13 \\ 3x+2y=12. \end{matrix}$ notice all the signs are + and the amounts are all different. To solve this type of problem (1) decide which letter you want to eliminate (2) then make the amounts the same and the signs different.

We will decide to eliminate y, we have 3y on the top line and 2y on the bottom line the common denominator of 3 and 2 is 6 so multiply the top line by 2 and the bottom line by - 3 this will make the amounts of y the same but the signs will be different .

## Word Problems and Simultaneous Equations

Many problems involving two things can be solved using simultaneous equations.

### *Method*

(1) Let $x$ = the **number** of the first item and $y$ = the **number** of the second item.

(2) Write out **two equations** in $x$ and $y$ using the information given in the question.

(3) Solve the equations to find $x$ and $y$.

### Example 1

If 7 pencils and 5 pens cost €1.16 and 5 pencils and 3 pens cost €0.76, find the cost of each.

Let $x$ = the price of a pencil and $y$ = the price of a pen.

Then $7x + 5y = 116$ ¢ and $5x + 3y = 76$ ¢.

Eliminate $y$ by multiplying the top line by –3 and the bottom line by 5. This gives

$$\begin{array}{lll} 7x+5y=116 & \times -3 & -21x-15y=-348 \\ 5x+3y=76 & \times 5 & 25x+15y=380 \\ & & \hline 4x \quad =32 \end{array}$$

$x = 8$. Now use this value of $x$ to find $y$.

$7x+5y=116$ becomes $7(8)+5y=116$, $56+5y=116$. $5y=60$. $y=12$

$\therefore$ a pencil costs 8 ¢ and a pen costs 12 ¢.

### Example 2

300 people came to a school play. Each adult paid €2.50 and each child paid €1.50. If each adult had been charged €3 and each child €1 there would have been an extra €24 taken in. How many adults attended?

Let $x$ = the number of adults and $y$ = the number of children.

Since 300 people came to the play $x + y = 300$. The takings in the first case are $250x + 150y$.

The takings using the new prices would be $300x + 100y$. The difference between the first and second takings is €24.

We can say that $250x + 150y + 2400 = 300x + 100y$.

If we tidy this up we get $50x - 50y = 2400$. which is the same as $x - y = 48$. We now have two equations:

$\begin{matrix} x+y=300 \\ x-y=48 \end{matrix}$. Adding we get $2x = 348$, $x = 174$ .so the number of adults is 174.

### Example 3

Only two candidates P and Q stood for election in which 850 votes were cast. P won the election However, if Q had taken 50 more votes from P she would have won the election by 20 votes. How many votes did each receive?

Let $x$ = number of P's votes and $y$ = number of Q's votes. Then $x + y = 850$.

The next line says that if Q took 50 votes from P she would end up with 20 more votes than P.

This can be written as follows: $x - 50 + 20 = y + 50$ , which gives $x - y = 80$. We now have our two equations

$\begin{matrix} x+y=850 \\ x-y=80 \end{matrix}$. Adding gives $2x = 930$, $x = 465$, $y = 385$.

$2x+3y=13.\times 2 \\ 3x+2y=12\times -3$ $\Rightarrow \begin{matrix} 4x+6y=26 \\ -9x-6y-36 \end{matrix}$ $add\ -5x=-10\Rightarrow x=2$ now find y we know 2x+3y = 13 and x= 2 , 2(2) + 3y = 13 , 3y = 9 y = 3 .

## Solving a linear equiation and a quadratic equation :

**Example 1 Solve x -3y + 10 = 0 ( linear) and $x^2 + y^2 = 10$ (quadratic).**

Method (1) Write the linear equation as either x = or y = . (2) substitute this value for x or y into the quadratic . (3) Simplify (4) Solve the quation (5) Use the value of x or y found in the linear equation to find the second variable .

(1) x-3y + 10 gives x = 3y + 10 . Now replace x in the quadratic by 3y + 10 to get

$(3y+10)^2+y^2=10\Rightarrow 9y^2+60y+100+y^2=10\Rightarrow 10y^2+60y+90=0$ Now find x . We know y

$y+6y+9=0\Rightarrow (y+3)(y+3)=0\Rightarrow y=-3.$

= - 3 and x = 3y +10 this means x = 3(-3) + 10 , x = 1 . The solution is x = 1 y = -3 .

This type of problem often occurs in the coordinate geometry of the circle question on the Leaving Cert Lower paper 2 .

**Example 2 : Solve x + 2y = 3 and $x^2 + y^2 = 26$ .**

(1) x = -2y + 3 : ( 2) sub this value of x into $x^2+y^2= 26$ to get

$(-2y+3)^2+y^2=26\Rightarrow 4y^2-12y+9+y^2=26\Rightarrow 5y^2-12y-17=0$

$(5y-17)(y+1)=0\Rightarrow y=-1, y=17/5.(y=3.4)$

$x=-2y+3, y=-1\Rightarrow x=-2(-1)+3\Rightarrow x=5. y=3.4\Rightarrow x=-2(3.4)+3\Rightarrow x=-3.8$

So our solution is (5,-1) and (-3.8, 3.4).

## Solving Cubic Equations :

The method for solving cubic equations is based on the **factor theorem** which states that **if x = a is a root of a particular equation then x - a is a factor of that equation .** In other words if you have a root of an equation then x- root is a factor .

Example 1 :sShow x = 5 is a root of $x^3-x^2-17x-15=0$ if x = 5 is a root the if we sub x = 5 into the equation we should get 0 = 0

$.(5)^3-(5)^2-17(5)-15\Rightarrow 125-25-85-15=0.\Rightarrow x=5$ is a root this implies x - 5 is a factor to find the other factor divide the cubic by x - 5 , the simplist method to the long division

5 1 -20 -17 -15

is to use synthetic division.....5...20...15 the second factor

.....4....3.....0

is $x^2+4x+3=(x+3)(x+1)$ now solve (x+3)(x+1) = 0 x = -3 and x = - 1 . so the three roots are 5,-3,-1 .

**Example 4 : Solve $2x^3-5x^2+x+2=0$. Find the first root by trial and error (usually a value of x between -3 and + 3 ). Try x = 1 $2(1)^3-5(1)^2+1+2=0\Rightarrow 2-5+1+2=0$ this implies that x = 1 is aroot . Now divide the cubic b y (x - 1) using Synthetic devision .**

**1] 2 - 5 + 1 + 2 our second factor is $2x^2$-3x -2 = ( 2x+ 1) (x -2) now solve (2x+1)(x-2) = 0**

**2 -3 -2 x = -1/2, x = 2 .**

**-3 -2 0**

## *Factors of quadratic expressions*

Expressions of the form $ax^2+bx+c$, $ax^2+bx$, $ax^2+b$ are called quadratic expressions.

*Example*: $3x^2+6x=3x(x+2)$

*Example*: $5x^2-20=5(x^2-4)=5(x-2)(x+2)$.

When it comes to factorising expressions of the form $ax^2+bx+c$ we need a good method. The following method works well.

There are 3 types of quadratic expressions in the form $ax^2+bx+c$.

**Type 1**: $ax^2+bx+c$ (all +, 2 plus factors).

**Type 2**: $ax^2-bx+c$ (– in the middle, + at the end, 2 minus factors).

**Type 3**: $ax^2+bx-c$ or $ax^2-bx-c$ (– at the end, 1 minus and 1 plus factor).

### Example 1: Type 1

Factorise $3x^2+7x+2$. All +, therefore 2 plus factors.

Write here the factors of $3x^2$

$3x^2+7x+2$

$(3x \quad +1)$

$(x \quad +2)$

Write here the factors of 2

$6x+x=7x$

Cross multiply to check middle is correct. $\therefore\ 3x^2+7x+2=(3x+1)(x+2)$.

### Example 2: Type 2

Factorise $(3x^2-7x+2)$.

Minus in the middle, plus at the end, $\therefore$ 2 minus factors.

$(3x^2-7x+2)$

Factors of $3x^2 \longrightarrow$ $3x \quad -1$ $\longleftarrow$ factor of 2

$x \quad -2$

$-6x-x=-7x$

Always check the middle. $(3x^2-7x+2)=(3x-1)(x-2)$.

### Example 3: Type 3

Factorise $3x^2-x-2$. Minus at the end, $\therefore$ 1 plus and 1 minus factor.

Always put the middle sign opposite the smaller number (works most of the time).

$3x^2-x-2$

$3x \quad +2$

$x \quad -1$

. Check the middle: $3x^2-x-2=(3x+2)(x-1)$.

$-3x+2x=-x$

### Example 4

Factorise $3x^2+2x-8$. Minus at the end, $\therefore$ 1 plus and 1 minus factor.

$3x^2+2x-8$

$3x \quad -4$

$x \quad +2$

Put the middle sign opposite the smaller number.

$6x-4x=2x$

$3x^2+2x-8=(3x-4)(x+2)$.

### Example 5

Factorise $10x^2 - 3x - 1$. Again 1 plus and 1 minus factor.

$10x^2 - 3x - 1$

$5x \nearrow \searrow +1$

$2x \searrow \nearrow -1$ Put the middle sign opposite the smaller number.

$-5x + 2x = -3x$

$10x^2 - 3x - 1 = (5x + 1)(2x - 1)$.

### Example 6

Sometimes it is necessary for a bit of trial and error as follows but practice will solve most of the problems:

| | |
|---|---|
| $24x^2 + x - 3$ | $24x^2 + x - 3$ |
| $6x \nearrow \searrow -3$ | $3x \nearrow \searrow -1$ |
| $4x \searrow \nearrow +1$ | $8x \searrow \nearrow +3$ |
| $6x - 12x = -6x \neq x$ | $-8x + 9x = x$ |

## Quadratic Equations

Quadratic equations are equation of the form $ax^2 + bx + c = 0, \quad a \neq 0, \quad a,b,c \in \mathbb{R}$.

Examples of quadratic equations:

$5x^2 + 11x + 2 = 0, \quad 5x^2 + 11x = 0, \quad 4x^2 - 9 = 0$

### *Method 1*

To solve a quadratic equation:

(1) Find the factors, (2) set each factor equal to zero, (3) solve the two new equations.

Note all quadratic equations have 2 solutions, called the roots of the equation.

#### Example 1

Solve $5x^2 + 11x + 2 = 0$

$\Rightarrow \quad (5x+1)(x+2) = 0$

$\Rightarrow \quad 5x + 1 = 0, \quad x + 2 = 0$

$5x = -1 \qquad x = -2$

$x = -\frac{1}{5}$

#### Example 2

Solve $5x^2 + 11x = 0$

$\Rightarrow \quad x(5x+11) = 0$

$\Rightarrow \quad x = 0 \quad \text{and} \quad 5x + 11 = 0 \quad \Rightarrow \quad 5x = -11$

$x = 0 \quad \text{and} \quad x = -\frac{11}{5}$

#### Example 3

Solve $4x^2 - 9 = 0$

$\Rightarrow \quad (2x-3)(2x+3) = 0$

$\Rightarrow \quad 2x - 3 = 0 \quad \text{and} \quad 2x + 3 = 0$

$2x = 3 \quad \text{and} \quad x = -\frac{3}{2}$

$x = \frac{3}{2}$

### *Method 2*

If you are unable to find the factors or the quadratic does not have any, use the following method.

The roots of $ax^2 + bx + c = 0$ can be found using the formula

$$x = \frac{-b \mp \sqrt{b^2 - 4ac}}{2a}$$

$a$ is the number in front of $x^2$, $b$ is the number in front of $x$ and $c$ is the independent term.

#### Example 4

Solve $x^2 + 3x + 1 = 0$

$a = 1, \quad b = 3, \quad c = 1$

$$\therefore \; x = \frac{-b \mp \sqrt{b^2 - 4ac}}{2a} = \frac{-3 \mp \sqrt{3^2 - 4(1)(1)}}{2(1)} \Rightarrow \quad x = \frac{-3 \mp \sqrt{5}}{2} = \frac{-3 \mp 2.236}{2}$$

$$= \frac{-3 + 2.236}{2}, \quad \frac{-3 - 2.236}{2} = -0.38, \quad -2.62$$

#### Example 5

Solve $3x^2 - 11x = 0$. Rewrite as $3x^2 - 11x + 0 = 0$

$a = 3, \quad b = -11, \quad c = 0$

$$x = \frac{-b \mp \sqrt{b^2 - 4ac}}{2a} \Rightarrow \frac{-(-11) \mp \sqrt{(-11)^2 - 4(3)(0)}}{2(3)} = \frac{11 \mp \sqrt{121}}{6} \text{ (be careful)}$$

$$= \frac{11 \mp 11}{6} = \frac{11 + 11}{6}, \quad \frac{11 - 11}{6} = \frac{22}{6}, \quad \frac{0}{6} = 3.67, \quad 0$$

#### Example 6

Solve $3x^2 - 10 = 0$. Rewrite as $3x^2 + 0x - 10 = 0$

$a = 3, \quad b = 0, \quad c = -10$

$$x = \frac{-b \mp \sqrt{b^2 - 4ac}}{2a} \Rightarrow \quad x = \frac{-(0) \mp \sqrt{0^2 - 4(3)(-10)}}{2(3)} = \frac{\mp\sqrt{120}}{6}$$

$$= \frac{\mp 10.95}{6} = \mp 1.83$$

$x = 1.83, \quad x = -1.83$

Ballinteer Institute
Holy Cross School Dundrum Dublin 14
Tel 01-2060006
Web: www.leavingcertsolutions.com

Email: John@projectmaths.com

# The Calculator file

**Casio83GT plus, Casio, fx-83ES**
**Sharp W531**

(i)Scientific Notation
(ii)Significant figures
(iii)Decimal Places,
(iv)Prime factors
(v)Percentages
(vi) Logs(LCH only)
(vii)Cartesian to polar form and back (LCH only)

## Statistics

(i )The Mean,
(ii)The Standard deviation from a frequency table
(iii)Correlation coefficient
(iv) The slope and intercept of the line best fit from a scatter plot.

Tables for functions (Casio)

# Casio 83GT Plus/Casio 83 ES
# Some additional uses

**Clearing Memory**

This is a good idea as any data stored will affect future calculations .

## To clear the contents of all memories

**Press Shift 9 3 = yes AC**

**Prime numbers**

**To find the prime number factors of a given number or write a number as a product of prime factors.**

**Example 1: write 2772 as a product of prime factors**

**2772 = then press shift ,,,,, (commas )(Fact ) =** $2^2 \times 3^2 \times 11$

**Example 2: Write 107525 as a product of prime factors input 107525 = shift ,,,,**

$= 5^2 \times 11 \times 17 \times 23$ .

**Example 3: Leaving Cert Ord Maths 2011 Question 1 Paper 1 ;**

**Write 2652 as a product of prime factors**

**2652= Shift ,,,, (commas)** $2^2 \times 3 \times 13 \times 17$ .

**Tables and functions**

**Example 1: if** $f(x) = 3x^2 - 5x - 2$ **:for** $-3 \le x \le 3$ **there are lots of ways to find the values of f(-3),f(-2) etc .**

**We can use the table function on your calculator as follows .**

**(i)Clear the memory using shift 9,3 = AC .**

**(ii)Mode 3 (table)** $f(x)$ **=**

**(iii)Press 3 alpha ,right bracket** $x^2$ **-5 alpha right bracket -2 = (iv)Start? Press -3 =**

**(v)End? Press 3 (vi)step? 1=A table will appear as follows**

| $x$ | $f(x)$ |
|---|---|
| $-3$ | 40 |
| $-2$ | 20 |
| $-1$ | 6 |
| 0 | $-2$ |
| 1 | $-1$ |
| 2 | 0 |
| 3 | 10 |

**For the function** $5x^3 - 4x^2 + 7x - 10$ **in the domain** $-2 \le x \le 3$

**We can use the table function on your calculator as follows .**

**(i)Clear the memory using shift 9,3 = AC .**

**(ii)Mode 3 (table)** $f(x)$ **= (iii)Press 5 alpha ,right bracket** $x^3$ **-4 alpha right bracket** $x^2$ **+7alpha right bracket -10 = (iv)Start? Press -2 = (v)End? Press 3 (vi)step? 1=A table will appear as follows (Vertically)**

$x$....$-2$....$-1$,.....0..1.....2.....3

$f(x)$.28,$-26$,$-10$,$-2$,..28,.110

## Changing from Rectangular coordinates to Polar Coordinates .

**Example 1:**To Change $1+\sqrt{3}i$ into polar Form

This will work in either Comp mode (mode 1) or STAT mode (mode 2)

(i)Press mode 1 .(ii)Shift + (Pol) [Pol C appears (iii)pres 1 shift ) right bracket , then $\sqrt{3}$ then = . The result is $r=2,\theta=60$ Therefore $1+\sqrt{3}i=2(Cos60+iSin60)$

**Example 2**: To Change $2+\sqrt{2}i$ into polar Form .

(i)Press mode 1 .(ii)Shift + (Pol) [Pol C appears (iii)pres 2 shift ) right bracket , then $\sqrt{2}$ then = . The result is $r=2.4495,\theta=35.2643$ Therefore

$2+\sqrt{2}i=2.4495(Cos35.2643+iSin35.2643)$

You can also change from Polar form to Carthesian (rectangular form )

Make sure you are in Degree mode (D) on Screen .

**Example 3;**

Change $2Cos135+iSin135$ into the form $a+bi$

(i) Mode 1.(ii)Shift – (minus sign Rec) Rec( appears on the screen .

(iii)press 2 then shift ) then 135 then equals the screen shows

$x=-1.4142=-\sqrt{2}, y=1.4142=\sqrt{2}\therefore 2Cos135+iSin135=-\sqrt{2}+\sqrt{2}i$

**Example 4:**

Change $4(Cos900+iSin900$ into the form $a+bi$

(i) Mode 1.(ii)Shift – (minus sign Rec) Rec( appears on the screen .

(iii)press 4 then shift ) then 900 then equals the screen shows

$x=-4, y=0\therefore 4(Cos900+iSin900)=4+0i$

**Scientific Notation :Significant figures:Decimal places**

Most students do not have a problem working with numbers in the form $ax10^{n}.\leq a<10, n\in Z$ (Scientific notation). They just use the $x10^{x}$ key on their calculator

**Example 1** Find the value of $1.34x10^{3}+2.68x10^{4}$ .Input into your calculator $1.34x10^{x}3+2.68x10^{x}4=28140$ .

Some questions require the answer in scientific notation .

So how do you do this ?

First clear the setup press shift 9,3,=AC. This is very important as the memory may not be empty..

Then press shift, mode , 7 (Sci) to get Sci 0-9?

Press any number between 0-9 to get your answer correct to that number of **significant figures** you require (if you do not it will default to 1significant figure).

Input $1.34x10^{x}3+2.68x10^{x}4=$ you will get $2.81400000x10^{4}$ (if you press 9 at the 0-9 stage ) and $2.814x10^{4}$ if you require 4 significant figures .

This key is also very useful for writing any number correct to a specific number of significant figures

**Example; 2** Write 1456000 correct to 3 significant figures. Press shift 9,3,= AC .Then shift mode 7(Sci) 0-9 press 3 then input $1456000=1.46x10^{6}$ .

**Example; 3** Write 0.0034567 correct to 4 significant figures . Press shift 9,3,= AC .Then shift mode 7(Sci) 0-9 press 4 then input $0.0034567=3.457x10^{-3}$

## Decimal places:

Press shift 9,3,= AC. Then shift mode 6(fix) 0-9 if you require 3 places press 3.

Example 4: $1.567 \times 2.896 = 4.538$ .Example 5 $0.0345 \div .00785 = 4.395$ correct to 3 decimal places.
**NOTE ALWAYS PRESS SHIFT 9 ,3,=,AC after using these keys!**

## Logarithms

The key $\log_a x$ key enables us to evaluate logs at different bases

Example 1: Evaluate $\log_3 27 = 3$.

Example 2: Evaluate $\log_4 (64)^{\frac{2}{3}} = 2$

Example 3: Evaluate $\log_3 27 + \log_2 32 + \log_4 64 = 11$

## Repeating Decimals fx-83 GT Plus

There is a key on the Casio 83 Gt ( not on the 83 Gs ) which will convert repeating decimals . The key is located above the $x^2$ key it looks like a box with a dot on top of it .

Example 1: Write 0.31313131..... as a fraction . Input 0.shift $x^2$ 31 $\rightarrow = \frac{31}{99}$,

Example 2; Write $1.\dot{5}\dot{6} = 1.56565656$ as a fraction .Input 1.shift $x^2$ 56 $\rightarrow = \frac{155}{99}$

Example 3: Write $123.1\dot{2}\dot{3} = 123.123123123$ as a fraction .Input 123.shift $x^2$ 123 $\rightarrow = \frac{41000}{333}$

**Casio fx-83GT/ESPlus calculator and statistics**

Given the frequency table find the mean and standard deviation

| X | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| f | 5 | 7 | 6 | 2 |

(i) press Mode 2,1 you will see a table as below imput the data as shown .Make sure the frequency is on by pressing shift mode (set up) down curser 1.

(ii)To get the mean press AC

(iii)Then shift 1.(iv) then 4 (on 83ES use 5) then 2 for the mean = 2.25.

(v)Then shift 1.and 3 for the standard deviation = 0.94207.

| | x | y |
|---|---|---|
| 1 | 1 = | 5= |
| 2 | 2 = | 7= |
| 3 | 3 = | 6= |
| 4 | 4 = | 2= |

Data from a scatter Plot

| X Score (independent variable) | Y score (Dependent Variable ) |
|---|---|
| 5 | 20 |
| 8 | 18 |
| 6 | 22 |
| 7 | 28 |
| 10 | 27 |

| | x | y | freq |
|---|---|---|---|
| 1 | 5 = | 20= | |
| 2 | 8 = | 18= | |
| 3 | 6 = | 22 = | |
| 4 | 7 = | 28 = | |
| 5 | 10 = | 27 = | |

(i) Press Mode 2 2 you will see a table as above imput the data as shown .

(ii) When all the data has been entered Press AC

(iii) Press shift 1 then press (5 (var) then press 3 (r= 0.4174 ) this will give the correlation coefficient

(iv)To get the slope of the regression line press shift 1 then 5 then 1 (A)=16.19 this is the intercept of the regression line and the Y axis .Press shift 1,5 then 2 (B= 0.9459) for the slope of the regression line. The regression line is in the form

For Casio Fx-83ES

(iii) Press shift 1 then press (7 (reg)then press 3 (r= 0.4174 ) this will give the correlation coefficient (iv)To get the slope of the regression line press shift 1 then 7(reg) then 1 (A)=16.19 this is the intercept of the regression line and the Y axis .Press shift 1,7(reg)then 2 (B= 0.9459) for the slope of the regression line. The regression line is in the form y-A-bx

# Casio Calculator

## Prime factors Significant figures Scientific Notation, Decimal places

Most students do not have a problem working with numbers in the form $ax10^n. \leq a < 10, n \in Z$ (Scientific notation). They just use the $x10^x$ key on their calculator

**Example 1** Find the value of $1.34x10^3 + 2.68x10^4$ .Input into your calculator 1.34 $x10^x3 + 2.68x10^x4 = 28140$ .Many questions require the answer in scientific notation.So how do you do this?

(i)clear memory press shift 9,3,=AC. This is very important.

(ii) press **shift, mode** , 7 (Sci) to get Sci 0-9?

(iii)Press any number between 0-9 to get your answer correct to that number of **Significant figures** if you do not it will default to 1significant figure. Input $1.34\,x10^x3 + 2.68x10^x4 =$ you will get $2.81400000\,x10^4$ (if you press 9 at the 0-9 stage ) and $2.814x10^4$ if you require 4 significant figures .This key is also very useful for writing any number correct to a specific number of significant figures

### Significant figures

**Example; 2** Write 1456000 correct to 3 significant figures. Press shift 9,3,= AC .Then shift mode 7(Sci) 0-9 press 3 then input 1456000 $= 1.46x10^6$ .

**Example;3** Write $0.0034567$ correct to 4 significant figures . Press shift 9,3,= AC .Then shift mode 7(Sci) 0-9 press 4 then input $0.0034567 = 3.457x10^{-3}$

### Decimal places:

Press shift 9,3,= AC. Then shift mode 6(fix) 0-9 if you require 3 places press 3.

**Example 4:** $1.567 \times 2.896 = 4.538$ .Example 5 $0.0345 \div .00785 = 4.395$ correct to 3 decimal places.

**NOTE ALWAYS PRESS SHIFT 9 ,3,=,AC after using these keys!**

# Sharp W531 Calculator and Statistics

The sharp w531 calculator can be used to find the **mean and standard deviation** as follows.

| X | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| f | 5 | 7 | 6 | 2 |

**Method**

**(i) Press mode then pres 0 the screen reads STAT 0 (SD).**

**(ii) Input data as follows 1 (x,y) 5 Data (change) the screen reads DATA SET = 1 .**

**(iii)Then 2(x,y) 7 Data etc the screen will read DATA SET =2**

**When all the data has been entered the screen will show DATA SET = 4.**

**(iv)To find the mean press RCL 4 ( $\bar{x}$ ) = 2.25**

**(v)To find the standard deviation press RCL 6 ( $\delta x$ ) = 0.94207**

To find the **Correlation coefficient** from a scatter plot using Sharp w 531 calculator.

| X Score (independent variable) | Y score (Dependent Variable ) |
|---|---|
| 5 | 20 |
| 8 | 18 |
| 6 | 22 |
| 7 | 28 |
| 10 | 27 |

**Method :**

**(i)Press mode then press 1 SCREEN READS STAT 1 (LINE).**

**(ii)Input data as follows 5(x,y) 20 data screen reads data set = 1 . (as above)**

**(iii)Repeat for the rest of the data .after the last imput screen should read DATA SET = 5.**

**(iv)To find the correlation coefficient press RCL ÷ (DIVISION SYMBOL) =0.417**

**(v)To find the equation of the regression line (line of best fit)**

**(vi)To find the slope of the regression line pres RCL ) (RIGHT BRACKET ) = .946**

**(vii)To find where the regression line cuts the y axis pres RCL ( (LEFT BRACKET) = 16.189**

**(viii)Therefore the equation of the line of best fit is** $y - 16.189 = .946x$

**(use** $y - y_1 = m(x - x_1)$ **the line of best fit is often written as** $y = a - bx$

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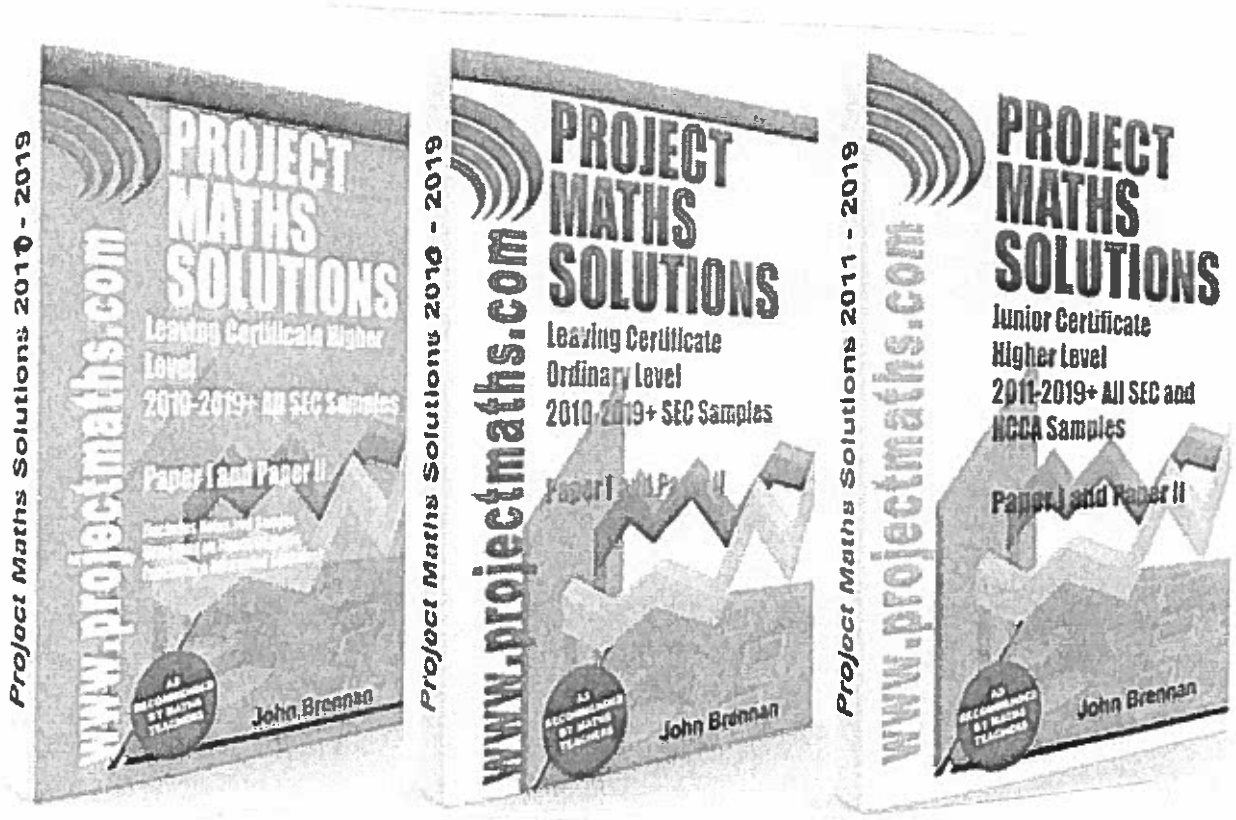
Tel 01-2960006
www.leavingcert.ie
email:john@projectmaths.com

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